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**Advanced Accelerator Applications  
Project Technical Note  
Research Project Office**

**Recuperator Analysis for the LBE Material Test  
Loop**



# Recuperator Analysis for the LBE Material Test Loop

**Document Number:**

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Revision 0

**Category: 1**

**Abstract:**

This document summarizes the calculations that were performed to determine the recuperator heat transfer capabilities and leakage flow through its ring seal.

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**Approval:**

Project Leader

AAA/RPO

**Signature on File – 9/4/2001**

\_\_\_\_\_  
Kemal Pasamehmetoglu / Date

# RECUPERATOR ANALYSIS COMMENT RESOLUTION SHEET

COMMENT	RESOLUTION	Initials
1. General comment: I think it would help to give sources for dimensions throughout. If all of the methods are correct, an error in a dimension would still give incorrect results.	Reference Added.	DJW
2. On p. 1, the third line from the bottom should say, "For the shell side" rather than "For the tube side."	Corrected.	DJW
3. On p. 2, the second number (8.55) in the first table will change based on the resistance correction for the tube wall discussed below for the Ap. A. This will also change the 36.6 kW and 48°C numbers in the paragraph below the first table.	Updated.	DJW
4. In the lower table on page 2, the second line should be 8.36E-4 rather than 7.75E-4 for the pipe size and velocity given.	Corrected.	DJW
5. On page 3 there is a typo in the legend for the figure (Between rather than Bewteen).	Corrected.	DJW
6. On page 3, the figure will need to be updated based on the correction for the tube wall discussed below for Ap. A.	Updated.	DJW
7. On page 3, the next to last sentence in the second paragraph should have 0.069 kg/s rather than 0.69 kg/s. The percentage given is correct.	Corrected.	DJW
8. On p. 3 of Ap. A, the stainless steel conductivity at 450°C is about 20.6 W/m-C. I think the 15.2 W/m-C given is close to room temperature. Using the 450°C value will help reduce the resistance through the wall.	Changed to 20.1 W/m°C based on the APT Materials Handbook values.	DJW
9. On p. 4 of Ap. A, the first and third resistances are based on 19 tubes, but the resistance through the tube wall is based on a single tube. Changing the resistance through the tube wall to also use that for 19 tubes will increase the power that can be transferred for the same inlet and exit temperatures.	Corrected.	DJW
10. I didn't check the cases you did in Ap. A for the range of flow conditions.	Noted.	DJW

<p>11. In Ap. B for the Ring Seal Leakage Analysis, on p. 1 I noticed that if you take 10 times the three lengths given on the bottom of the first page, you get 0.05 m. The figure above that shows a total of 0.055 m. I assume a short straight section on each end was not included that gives the 0.005 m difference.</p>	<p>Your assumption is correct.</p>	<p>29w</p>
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## Distribution List

Ammerman, Curtt	H821
APT-RMDC	C341
APT-RPO	H816
Pasamehmetoglu, Kemal	H816
Quintana, Lawrence (QA)	H809
Smith, Brian	H821
Tomei, Tony	H836
Woloshun, Keith	H855



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## Recuperator Analysis for the LBE Material Test Loop

### Introduction

The recuperator is a shell-and-tube model that is used to transfer heat between two flowing lead-bismuth eutectic (LBE) streams. Heat is transferred from the hot-side LBE prior to entering the heat exchanger, which reduces the heat removal load on the heat exchanger. Heat is transferred to the cold-side LBE prior to entering the heat input section, which reduces the load required in the heat input section. The recuperator design was supplied by the Russians. It was modified slightly by project engineers and designers and then fabricated in the U.S.

This document summarizes the calculations that were performed to determine the recuperator heat transfer capabilities and leakage flow through its ring seal.

### Recuperator Overview

A sectional view of the recuperator is shown in Fig. 1. The recuperator is essentially a shell and tube heat exchanger. The shell is constructed of 4 schedule 80 pipe (dimensions were obtained from the ESA-DE Recuperator Assembly drawings: #142Y600923). Within the shell are 19 tubes, each of which have a diameter of 9/16" and a wall thickness of 0.035". The tube bundle is fixed at one end of the shell, however to allow for differences in thermal expansion rates, the opposite end of the tube bundle is allowed to float. The seal on this floating end is composed of 10 sliding rings, similar to piston rings in an internal combustion engine.



**Fig. 1. Sectional View of Recuperator.**

The recuperator is configured in a counter-flow arrangement as shown in Fig. 1. The cold LBE flows on the shell side and the hot LBE flows on the tube side.

### Heat Transfer Analysis

An analysis was performed to determine the heat transfer capability of the recuperator. Details of the analysis are shown in Appendix A. The operating condition examined was:

Mass flow rate (both hot and cold sides):	5.25 kg/s
Hot side inlet temperature (tube side):	500°C
Cold side inlet temperature (shell side):	400°C

For the tube side heat transfer, fully developed turbulent flow was assumed with a uniform wall heat flux. The Nusselt number correlation provided by Reed [1] was used. For the shell side heat transfer, fully developed turbulent flow through a tube bundle was assumed. Several correlations were investigated, however the Nusselt number correlation suggested by Rehme [1] appeared to have the best support and, therefore, was used.

The table below shows the results of the thermal resistances calculated relative to the convection resistance inside the tubes:

Convection from LBE to tube wall	1.00
Conduction through 0.89-mm-thick tube wall	0.34
Convection from tube wall to LBE in shell	0.89

The controlling thermal resistance is the convection on the inside of the tubes. The result of this analysis shows a heat transfer rate from hot to cold LBE of 62.3 kW. The corresponding temperature drop on the hot side (and the corresponding temperature rise on the cold side) was 81°C.

### Heat Transfer Analysis of Alternate Flow Conditions

To provide an assessment of how the recuperator will perform at other operating conditions, a range of alternate flow conditions was examined. Three different volume flow rates and four different LBE inlet temperature conditions were combined resulting in 12 different off-design cases. The four inlet temperature conditions examined were specified as a temperature difference between the hot inlet and cold inlet. These temperature differences are 50°C, 100°C, 150°C, and 200°C. The volume flow rates examined are shown below along with their corresponding sources:

$3.00 \times 10^{-4} \text{ m}^3/\text{s}$	0.56 m/s in 1 schedule 40 pipe
$8.36 \times 10^{-4} \text{ m}^3/\text{s}$	1.5 m/s in 1 schedule 40 pipe
$4.33 \times 10^{-3} \text{ m}^3/\text{s}$	2.0 m/s in 2 schedule 40 pipe

To simplify the analysis, LBE properties were evaluated at the same conditions as those for the original analysis shown in Appendix A. A summary of the alternate flow condition analysis are also shown in Appendix A.



The results of the alternate flow condition analysis are shown in Fig. 2. Figure 2 shows a plot of hot-side (or cold-side)  $\Delta T$  versus LBE mass flow rate, for varying inlet temperature differences. This plot shows that the recuperator performance is a strong function of the temperature difference between the two flowing LBE streams.

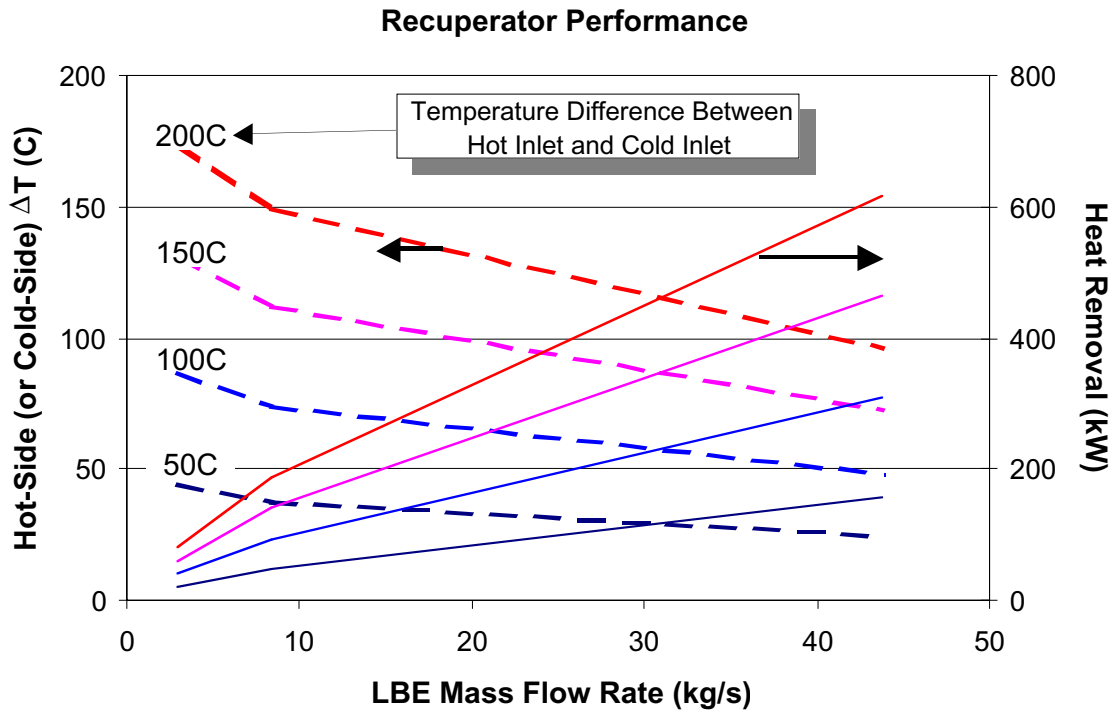


Fig. 2. Results of Alternate Flow Conditions.

### Leakage Flow Through Ring Seal

An analysis was performed to determine the leakage through the ring seal which separates the shell side from the tube side of the recuperator. The details of this analysis are shown in Appendix B. As mentioned previously, this seal is constructed from 10 sliding rings. Based on a loop pressure drop analysis performed by Valentina Tcharnotskaia, the pressure difference across the rings for a typical operating flow rate of 10 kg/s is approximately 414 kPa. This is the driving potential for the LBE leakage between the hot and cold sides.

The losses for the analysis were estimated using the correlations from Idelchik [2]. The rings were assumed to float in the center of the ring grooves allowing flow to pass freely around them. To simplify the analysis, the flow paths were modeled as flow between flat plates instead of flow between concentric cylinders. Microsoft Excel was used to solve for the leakage flow rate through 10 rings that corresponded to the 414 kPa driving



potential. The resulting leakage rate was 0.069 kg/s or 0.69%. This leakage flow rate is negligible.

### **Internal Pressure**

A room temperature, hydrostatic pressure test was performed on the recuperator to a pressure of 250 psig.

**References**

- [1] *Handbook of Single-Phase Convective Heat Transfer*, edited by Kacak, S., Shah, R.K., and Aung, W., Wiley, New York, 1987.
- [2] Idelchik, I.E., *Handbook of Hydraulic Resistance*, 3<sup>rd</sup> Edition, Begell House, New York, 1996.

## **Appendix A**

# **Recuperator Heat Transfer Analysis**

2/7/00 - Recuperator Analysis

①/4

<u>Hot Side (Tubes)</u>	<u>Cold Side (Shell)</u>
$\dot{V} = 5.17 \times 10^{-4} \text{ m}^3/\text{s}$	$\dot{V} = 5.17 \times 10^{-4} \text{ m}^3/\text{s}$
$T_{in} = 500^\circ\text{C}$	$T_{in} = 400^\circ\text{C}$
$T_{out} = 450^\circ\text{C}$	$T_{out} = 450^\circ\text{C}$
@ 475°C $\rho = 10150 \text{ kg/m}^3$	@ 425°C $\rho = 10211$
$\dot{V} = 1.41 \times 10^{-7} \text{ m}^3/\text{s}$	$\dot{V} = 1.52 \times 10^{-7}$
$k = 14.405 \text{ W/mK}$	$k = 13.94$
$Pr = 0.0146$	$Pr = 0.0163$
$C_p = 1464 \text{ J/kg}^\circ\text{C}$	$C_p = 1464 \text{ J/kg}^\circ\text{C}$

► Tube Side

$$19 \text{ tubes, } \frac{9}{16}'' \text{ OD, } 0.035'' \text{ wall} \Rightarrow 0.493'' \text{ ID} = 1.25 \times 10^{-2} \text{ m}, \left( \frac{L}{D} = \frac{116}{1.25} = 92.8 \right)$$

$$A_c = (19) \frac{\pi}{4} (1.25 \times 10^{-2} \text{ m})^2 = 2.34 \times 10^{-3} \text{ m}^2$$

$$A_s = 19 \pi (1.43 \times 10^{-2}) (1.16) = 0.989 \text{ m}^2$$

$$V = \frac{\dot{V}}{A_c} = \frac{5.17 \times 10^{-4} \text{ m}^3/\text{s}}{2.34 \times 10^{-3} \text{ m}^2} = 0.22 \text{ m/s}$$

$$Re_D = \frac{(0.22)(1.25 \times 10^{-2})}{(1.41 \times 10^{-7})} = 19504 \quad (Pe = Re Pr = 285)$$

A uniform wall heat flux (ala Reed) and uniform wall T ala Reed

$$Nu_H = 5.0 + 0.025 Pe_m^{0.8} = 7.30$$

$$Nu_T = 3.3 + 0.02 Pe_m^{0.8} = 5.14$$

use  $Nu_H$  and neglect thermal entry region

$$h = \frac{k}{D} Nu = \frac{(14.41)}{(1.25 \times 10^{-2})} (7.3) = \underline{\underline{8415 \text{ W/m}^2\text{K}}}$$

Shell side

$\frac{2}{4}$

- Outer tube: 114.3 mm OD, 8.56 mm wall  $\Rightarrow A_c = 7.42 \times 10^{-3} \text{ m}^2$  ( $ID = 9.72 \times 10^{-2} \text{ m}$ )
- Cross-section of Tubes:  $= (19) \frac{\pi}{4} (1.423 \times 10^{-2} \text{ m})^2 = 3.046 \times 10^{-3} \text{ m}^2$

• flow area  $= 4.37 \times 10^{-3} \text{ m}^2$

$$V = \frac{\dot{V}}{A_c} = \frac{5.17 \times 10^{-4} \text{ m}^3/\text{s}}{4.37 \times 10^{-3} \text{ m}^2} = 0.12 \text{ m/s}$$



$$D_h = \frac{4A}{P} = \frac{4(4.37 \times 10^{-3} \text{ m}^2)}{(\pi)(9.72 \times 10^{-2}) + 19\pi(1.423 \times 10^{-2})} = 1.51 \times 10^{-2} \text{ m}$$

$$Re_{Dh} = \frac{(0.12)(1.51 \times 10^{-2})}{(1.52 \times 10^{-7})} = 11921 \quad (Pe = 194)$$

- ~~extrapolating data of Dwyer (Foust p.113)~~

~~$$h \approx 19000 \frac{\text{Btu}}{\text{hr ft}^2 \cdot \text{F}} = 107962 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} ?$$~~

- using Sieder's (Foust p.113-114)

$$Nu = 0.58 \left( \frac{D_E}{D} \right)^{0.55} Pe^{0.45} \quad \frac{D_E}{D} = \frac{2\sqrt{3}}{\pi} \left( \frac{P}{D} \right)^2 - 1$$

$$\therefore Nu = (0.58)(0.323)^{0.55} (194)^{0.45} = 3.33$$

$$h = \frac{k}{D} Nu = \frac{(394)}{(1.51 \times 10^{-2})} (3.33) = 3079 \text{ W/m}^2 \cdot \text{K}$$

- ~~assuming flow in annulus using Kern's & Leung~~

~~$$Nu \approx 11.8 \Rightarrow h \approx 10934 ?$$~~

- ~~using Kottowski for flow across tube bundles~~

~~$$Nu = Pe^{0.5} = 13.93 \Rightarrow h = 12858$$~~

- using Maroasca & Dwyer's chart  $\Rightarrow Nu \approx 12-14$

Kacac

③/4

• Fully developed H.T, liquid metals, through tube bundles

$$Nu = Nu_{lam} + \frac{3.67}{90(P/D)^2} \left[ 1 - \frac{1}{\frac{1}{6}[(P/D)^{30} - 1] + \sqrt{1.24\epsilon_k + 1.15}} \right] Pe^{m_1}$$

$$m_1 = 0.56 + 0.19 \left( \frac{P}{D} \right) - 0.1 \left( \frac{P}{D} \right)^{-0.80} = 0.783$$

$$\epsilon_k = \frac{k_2}{k_3} \frac{1 - \Delta_0 (r_1/r_2)^2}{1 + \Delta_0 (r_1/r_2)^2}$$

$$\Delta_0 = \frac{k_2 - k_1}{k_2 + k_1}$$

$$k_1 = k_{fuel}$$

$$k_2 = k_{cladding}$$

$$k_3 = k_{fluid}$$

$r_1, r_2$  = inner, outer radii of cladding

$$k_2 = k_{ss} = 20.1 \text{ W/m}^2\text{K}$$

$$r_1 = 6.25 \times 10^{-3} \text{ m}$$

$$r_2 = 7.12 \times 10^{-3} \text{ m}$$

$$k_1 = \infty \text{ At } T = \text{constant inside tube}$$

$$k_3 = 13.94 \text{ W/mK}$$

$$\Delta_0 \approx -1$$

$$\epsilon_k = \frac{15.2}{13.94} \frac{1 + \left( \frac{6.25}{7.12} \right)^2}{1 - \left( \frac{6.25}{7.12} \right)^2} = 1.67$$

$$Nu_{lam} = \left( 7.55 \frac{P}{D} - \frac{6.3}{(P/D)^{17(P/D)(P/D - 0.81)}} \right) \left( 1 - \frac{3.6(P/D)}{(P/D)^{30}(1 + 2.5\epsilon_k^{0.36}) + 3.2} \right)$$

$$Nu_{lam} = (7.55)(0.9777) = 7.41$$

$$Nu = 7.41 + 0.0283 \left( 1 - \frac{1}{40.82} \right) (194)^{0.788} = 9.16$$

$$h = \frac{k}{D_h} Nu = \frac{13.94}{1.51 \times 10^{-2}} (9.16) = \underline{\underline{8456 \text{ W/m}^2\text{K}}}$$

④/4

### Thermal Resistances

$$R_{th1} = \frac{1}{hA} = \frac{1}{(19)h\pi DL}$$

$$h = 8415 \text{ W/m}^2\text{C}, D = 1.25 \times 10^{-2} \text{ m} \quad (L = 1 \text{ m})$$

$$R_{th1} = 1.59 \times 10^{-4} \frac{\text{m}^2\text{C}}{\text{W}}$$

$$R_{th2} = \frac{\ln(r_o/r_i)}{2\pi k L(19)}$$

$$D_o = 1.423 \times 10^{-2} \text{ m}, D_i = 1.25 \times 10^{-2} \text{ m}, K_{450^\circ\text{C}} = 20.1 \text{ W/m}^2\text{C}$$

$$R_{th2} = 5.40 \times 10^{-5} \frac{\text{m}^2\text{C}}{\text{W}}$$

$$R_{th3} = \frac{1}{hA} = \frac{1}{(19)h\pi DL}$$

$$h = 8456 \text{ W/m}^2\text{C}, D = 1.423 \times 10^{-2} \text{ m}$$

$$R_{th3} = 1.39 \times 10^{-4} \frac{\text{m}^2\text{C}}{\text{W}}$$

$$\frac{UA}{L} = \frac{1}{R_{th,tot}} = 2841 \frac{\text{W}}{\text{m}^2\text{C}}$$

$$A \text{ heat transfer length} = 45.5'' = 1.16 \text{ m}$$

### Solution

$$\left. \begin{aligned} q_1 &= \dot{m}_h c_{ph} (T_{hi} - T_{ho}) \\ q_2 &= \dot{m}_c c_{pc} (T_{co} - T_{ci}) \\ q_3 &= UA \Delta T_{LMTD} \end{aligned} \right\} q_1 = q_2 = q_3$$

Solve simultaneously  
using MS Excel

$$Q = 62.3 \text{ kW}$$

$$\Delta T = 81^\circ\text{C}$$

L	1.16		
ua	3295.56		
thi	500		
tci	400	q1	62379
cph	146.6	q2	62379
cpc	146.6	q3	62379
mh	5.248		
mc	5.249		
tho	418.9		
tco	481.1		



Examine Alternate Flow Conditions

- 1)  $3.00 \times 10^{-4} \text{ m}^3/\text{s}$  (0.54 m/s in 1" sch. 40 pipe)  
 2)  $8.36 \times 10^{-4} \text{ m}^3/\text{s}$  (1.5 m/s in 1" sch. 40 pipe)  
 3)  $4.33 \times 10^{-3} \text{ m}^3/\text{s}$  (2.0 m/s in 2" sch. 40 pipe)

- $h_{\text{tube}} (\text{W/m}^2\text{C})$   
 1) 7479  
 2) 9670  
 3) 20,324

- $h_{\text{shell}} (\text{W/m}^2\text{C})$   
 7771  
 9180  
 15,293

} use properties from original analysis

- 1)  $\frac{UA}{L} = 2599 \frac{\text{m}^2\text{C}}{\text{W}}$   
 2)  $\frac{UA}{L} = 3117 \frac{\text{m}^2\text{C}}{\text{W}}$   
 3)  $\frac{UA}{L} = 5077 \frac{\text{m}^2\text{C}}{\text{W}}$

• combine flow conditions w/ temperature variations

⇒ temperature difference between hot inlet and cold inlet

- a) 50°C  
 b) 100°C  
 c) 150°C  
 d) 200°C

• solve the resulting matrix in Excel for Q and  $\Delta T$

$\Delta T (^\circ\text{C})$	50°C	100°C	150°C	200°C
$3.00 \times 10^{-4} \text{ m}^3/\text{s}$	44	87	131	174
$8.36 \times 10^{-4} \text{ m}^3/\text{s}$	37	74	112	149
$4.33 \times 10^{-3} \text{ m}^3/\text{s}$	24	48	72	96

Q (kW)	50°C	100°C	150°C	200°C
$3.00 \times 10^{-4} \text{ m}^3/\text{s}$	19.4	38.9	58.3	77.8
$8.36 \times 10^{-4} \text{ m}^3/\text{s}$	46.3	92.6	138.8	185.1
$4.33 \times 10^{-3} \text{ m}^3/\text{s}$	153.8	307.7	461.5	615.4

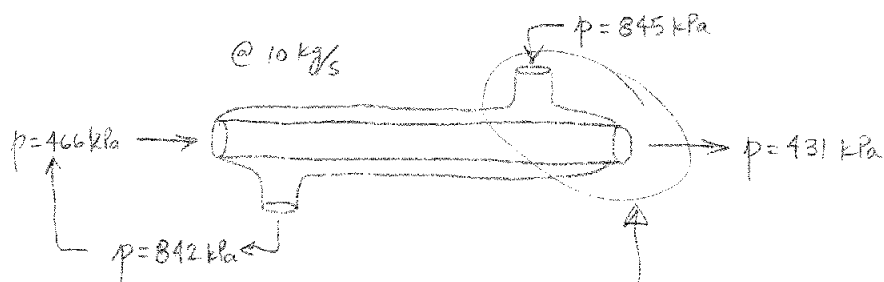
## **Appendix B**

### **Ring Seal Leakage Analysis**

①/2

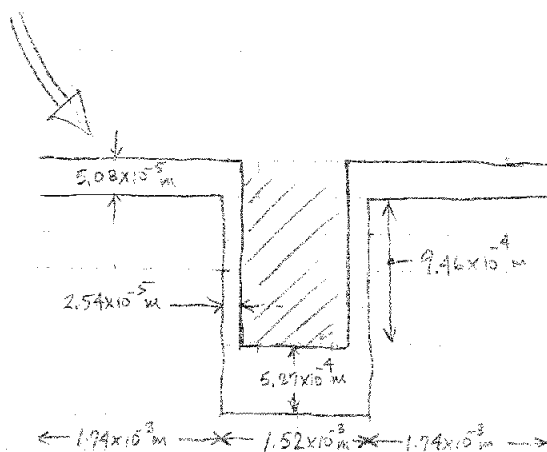
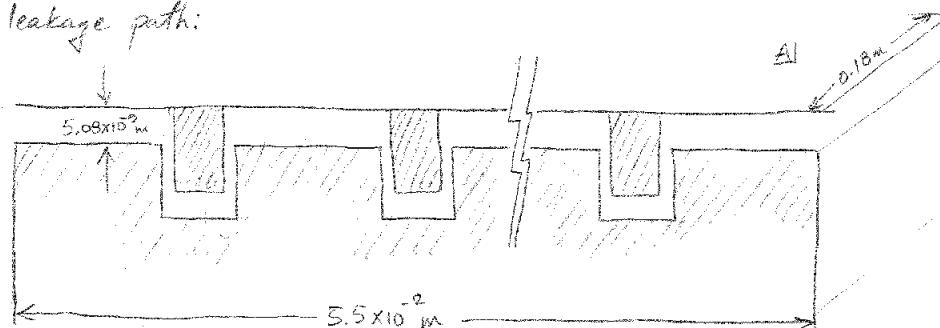
# Recuperator Ring Leakage Calc

$\Delta p$  calc from Valentina's spreadsheet:



• driving  $\Delta p$  for leakage =  $414 \text{ kPa}$

• leakage path:



$\frac{2}{2}$ 

Use Idetchik for each of the 4,  $90^\circ$ 's (diagram 6-6)

$$1) D_n = \frac{2(5.08 \times 10^5)(0.18)}{5.08 \times 10^5 + 0.18} = 1.02 \times 10^{-4} \text{ m}$$

$$A/R_{\text{down}} \geq 2 \times 10^5$$

A  $T \approx 425^\circ\text{C}$

$\rho = 10211 \frac{\text{kg}}{\text{m}^3}$

$$V = 1.52 \times 10^{-7} \text{ m}^3/\text{s}$$

$$\frac{a_0}{b_0} \sim \infty, \quad \frac{b_1}{b_0} = 0.5 \Rightarrow k_1 \approx 1.8$$

$$2) \quad \frac{a_0}{b_0} \sim \infty, \quad \frac{b_1}{b_0} \approx 21 \Rightarrow K_2 \approx 0.55$$

$$D_h = \frac{2(2.54 \times 10^{-5})(0.18)}{(2.54 \times 10^{-5}) + 0.18} = 5.08 \times 10^{-5} \text{ m}$$

3)  $\frac{b_1}{b_0} \approx 0.05$   $K = 0.5 \text{ a (diag 4.9)} \Rightarrow K_3 \approx 1.0$   $D_h = 1.05 \times 10^3 \text{ m}$

4)  $\frac{b_1}{b_2} \approx 2 \Rightarrow k_3 \approx 0.55$   $D_h = 5.08 \times 10^{-5} \text{ m}$

► Use Idelchik for friction in each section (diag. 2-6) A)  $Re_D = 2000$

$$K_{1f} = 1.5 \frac{1}{D_h} = 1.5(0.032) \frac{1.74 \times 10^3}{1.02 \times 10^{-4}} = 0.82$$

$$K_{2f} = 1.5 (0.032) \frac{9.46 \times 10^{-4}}{5.08 \times 10^{-5}} = 0.89$$

$$K_{zf} = 15 (0.032) \frac{1.05 \times 10^3}{1.52 \times 10^{-3}} = 0.63$$

$$K_{4f} = K_{2f} = 0.89$$

$$K_{sf} = K_{lf} = 0.82$$

Use Excel to solve for flow rate that matches  $\Delta p = 414 \text{ kPa}$

	dh	l	d	w	vel	k	Dp	
1	1.02E-04	1.74E-03	5.08E-05	0.18	0.74	2.62	7246	
2	5.08E-05	9.46E-04	2.54E-05	0.18	1.47	1.44	15930	
3	1.05E-03	1.52E-03	5.27E-04	0.18	0.07	1.03	26	
4	5.08E-05	9.46E-04	2.54E-05	0.18	1.47	1.44	15930	
5	1.02E-04	1.74E-03	5.08E-05	0.18	0.74	0.82	2268	
							414000	414000
			vdot	6.73E-06				
			vdot%	0.69				
			mdot	0.0687	0.006872			